MATHEMATICS

COMPACTIFICATIONS IN WHICH THE COLLECTION OF MULTIPLE POINTS IS LINDELÖF SEMI-STRATIFIABLE

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ABSTRACT

We show that every compactification of a topological space in which the collection of multiple points is Lindelöf semi-stratifiable is a z-compactification. In particular such a compactification is a Wallman compactification.

KEY WORDS AND PHRASES

Wallman compactification, z-compactification, semi-stratifiable, βX.

1. INTRODUCTION

Every Tychonoff space X admits Hausdorff compactifications, obtainable as the ultra-filter space of some normal base on X. These compactifications are called *Wallman compactifications*. Until now the question, raised in [2] and [6], whether all Hausdorff compactifications are Wallman compactifications remains unanswered, although many well known compactifications turned out to be Wallman compactifications ([1], [4], [9], [10], [11], [12]).

In this paper we will show that any Hausdorff compactification of a topological space in which the collection of multiple points is Lindelöf semi-stratifiable is a z-compactification (a compactification obtainable as the ultra-filter space of a normal base consisting of zero-sets of X). This theorem generalizes results obtained by Aarts ([1]), Steiner and Steiner ([9], [11]) and Berney ([4]). Moreover we will show that every Hausdorff compactification of a Lindelöf semi-stratifiable space, in which the collection of multiple points also is Lindelöf semi-stratifiable, is *regular Wallman* (in the sense of Steiner ([12]); such a space is a Wallman compactification of each dense subspace).

Most of the results in this paper are also to be found (in a preliminary form) in the report [7a].

2. WALLMAN COMPACTIFICATIONS

All topological spaces, under discussion, are assumed to be Hausdorff. Let X be a topological space and let $\mathscr{S}$ be a collection of subsets of X. We will write $\vee \cdot \mathscr{S}$ for the family of finite unions of elements of $\mathscr{S}$ and

$\wedge\cdot\mathscr{S}$ for the family of finite intersections of elements of $\mathscr{S}$. The family $\wedge\cdot\vee\cdot\mathscr{S}=\vee\cdot\wedge\cdot\mathscr{S}$ is closed both under finite unions and intersections; it is called the *ring* generated by $\mathscr{S}$. We say that $\mathscr{S}$ is *separating* if for each closed subset $F\subset X$ and for each $x\in X\backslash F$ there exist S_0, $S_1\in\mathscr{S}$ such that $x\in S_0$, $F\subset S_1$ and $S_0\cap S_1=\emptyset$. The collection $\mathscr{S}$ is called *normal* if for each $S_0, S_1\in\mathscr{S}$ with $S_0\cap S_1=\emptyset$ there exist $T_0, T_1\in\mathscr{S}$ such that $S_0\cap T_1=\emptyset=T_0\cap S_1$ and $T_0\cup T_1=X$. A *normal base* is a normal separating ring consisting of closed subsets of X. A compactification αX of X is a Wallman compactification iff αX possesses a separating family $\mathscr{F}$ of closed subsets such that for all $F_i\in\mathscr{F}$ $(i=1, 2, \ldots, n; n\in N)$ with $\bigcap_{i=1}^n F_i\neq\emptyset$, we have $\bigcap_{i=1}^n F_i\cap X\neq\emptyset$. (Steiner ([12])). A compactification αX of X is called a z-compactification if $\mathscr{F}$ in the above characterization can be chosen in such a way that for all $F\in\mathscr{F}$ the set $F\cap X$ is a zero-set in X. A topological space X is called *strongly* $\aleph$ *compact* if for each subset A of X with card $(A)\geqslant\aleph$ and for each total order $<$ on A there exists a $y\in A$ such that for each open neighbourhood U of y both $U\cap\{x\in A|x<y\}$ and $U\cap\{x\in A|x>y\}$ are non void. It is very easy to show that each separable metric space is strongly $\aleph_1$ compact using the fact that such a space admits a countable base. More general Creede ([5]) showed that *each Lindelöf semi-stratifiable space is strongly* $\aleph_1$ *compact* (a topological space X is *semi-stratifiable* if to each open set $U\subset X$, one can assign a sequence $\{U_n\}_{n=1}^\infty$ of closed subsets of X such that (a) $\bigcup_{n=1}^\infty U_n=U$, (b) $U_n\subset V_n$ whenever $U\subset V$ (where $\{V_n\}_{n=1}^\infty$ is the sequence assigned to V)). Moreover *strongly* $\aleph_1$ *compact spaces are hereditarily seperable and hereditarily Lindelöf* (Berney ([3])). E. S. Berney has introduced the concept of strongly $\aleph$ compactness in the theory of Wallman compactifications. His techniques turn out to be very powerfull and will be used in this paper.

Let αX be a compactification of X and let ξ denote the unique projection map of βX, the Čech-Stone compactification of X, onto αX which on X is the identity. We say that a point $p\in\alpha X\backslash X$ is a *multiple point* of αX if $\xi^{-1}(p)$ consists of more than one point. Let $Z(X)$ denote the ring of zero-sets of X.

LEMMA 2.1: *Let* Y *be a subspace of* βX *such that* $X\subset Y\subset\beta X$. *If* $Z_0, Z_1\in Z(X)$ *then* $\mathrm{cl}_Y(Z_0)\cap\mathrm{cl}_Y(Z_1)=\mathrm{cl}_Y(Z_0\cap Z_1)$.

PROOF: $\mathrm{cl}_{\beta X}(Z_0)\cap\mathrm{cl}_{\beta X}(Z_1)=\mathrm{cl}_{\beta X}(Z_0\cap Z_1)$. $\square$

For the remainder of this section, let αX be a compactification of the topological space X and let M denote the set of mutiple points of αX.

LEMMA 2.2: *Let* $Z\in Z(X)$. *If* $\eth\,\mathrm{cl}_{\alpha X}(Z)\cap M=\emptyset$, *then* $\xi^{-1}(\mathrm{cl}_{\alpha X}(Z))=$ $=\mathrm{cl}_{\beta X}(Z)$.

PROOF: Assume there exists an

$$x_0\in\xi^{-1}(\mathrm{cl}_{\alpha X}(Z))\backslash\mathrm{cl}_{\beta X}(Z).$$

Then $\xi(x_0) \in \mathrm{cl}_{\alpha X}(Z) \cap M$ and consequently $\xi(x_0) \in \mathrm{int}_{\alpha X}\, \mathrm{cl}_{\alpha X}(Z)$ since $\partial\, \mathrm{cl}_{\alpha X}(Z) \cap M = \emptyset$. Therefore

$$x_0 \in \xi^{-1}(\mathrm{int}_{\alpha X}\, \mathrm{cl}_{\alpha X}(Z)) \subset \mathrm{int}_{\beta X}\, \xi^{-1}(\mathrm{cl}_{\alpha X}(Z)).$$

Let 0 be an open neighbourhood of x_0 in βX. Then

$$0 \cap \mathrm{int}_{\beta X}\, \xi^{-1}(\mathrm{cl}_{\alpha X}(Z)) \neq \emptyset$$

so that

$$0 \cap \mathrm{int}_{\beta X}\, \xi^{-1}(\mathrm{cl}_{\alpha X}(Z)) \cap X \neq \emptyset.$$

As ξ is the identity on X it follows that $0 \cap Z \neq \emptyset$. Therefore $x_0 \in \mathrm{cl}_{\beta X}(Z)$, which is a contradiction. □

If $f \in C(\alpha X, I)$ then we will write $U(\delta, f)$ instead of $f^{-1}([0, \delta))$.

Lemma 2.3: *Let $f \in C(\alpha X, I)$ and assume that M is strongly $\aleph_1$ compact. Then* card $\{\delta \in (0,1) | \mathrm{cl}_{\alpha X}(f^{-1}[0,\delta] \cap X) \cap M \neq \mathrm{cl}_{\alpha X}(U(\delta, f)) \cap M\} \leqslant \aleph_0$.

Proof: Assume to the contrary that

$$\mathrm{card}\, \{\delta \in (0,1) | \mathrm{cl}_{\alpha X}(f^{-1}[0,\delta] \cap X) \cap M \neq \mathrm{cl}_{\alpha X}(U(\delta, f)) \cap M\} \geqslant \aleph_1.$$

If $\mathrm{cl}_{\alpha X}(U(\delta, f)) \cap M \neq \mathrm{cl}_{\alpha X}(f^{-1}[0,\delta] \cap X) \cap M$ then there exists an

$$a(\delta) \in (\mathrm{cl}_{\alpha X}(f^{-1}[0,\delta] \cap X) \backslash \mathrm{cl}_{\alpha X}(U(\delta, f))) \cap M.$$

Let B be the collection of $a(\delta)$ chosen in this way. Since $f(a(\delta)) = \delta$ it follows that $\delta_1 \neq \delta_2$ implies that $a(\delta_1) \neq a(\delta_2)$ and therefore B is uncountable. Also, a total order "$<$" is defined on B by putting $a(\delta_0) < a(\delta_1) \Leftrightarrow \delta_0 < \delta_1$. Since $B \subset M$ and since M is strongly $\aleph_1$ compact it follows that B has a limit point $a(\delta_0)$ from below. Let U be an arbitrary open neighbourhood of $a(\delta_0)$. Since $a(\delta_0)$ is a limit point from below there is an $a(\delta_1) \in U \cap B$ such that $a(\delta_1) < a(\delta_0)$. This shows that $a(\delta_1) \in U(\delta_0, f) \cap U$ and in particular $U \cap U(\delta_0, f) \neq \emptyset$. Hence $a(\delta_0) \in \mathrm{cl}_{\alpha X}(U(\delta_0, f)) \cap M$, which is a contradiction. □

Lemma 2.4: *Let $f \in C(\alpha X, I)$ and let U be open in αX. If $A \subset \alpha X$ is strongly $\aleph_1$ compact then*

$$\mathrm{card}\, \{\delta \in (0,1) | \mathrm{cl}_{\alpha X}(U) \cap \mathrm{cl}_{\alpha X}(U(\delta, f)) \cap A \neq \mathrm{cl}_{\alpha X}(U \cap U(\delta, f)) \cap A\} \leqslant \aleph_0.$$

Proof: Use the same technique as in lemma 2.3 or see Berney ([4]). □

Theorem 2.6: *Any compactification of a topological space X in which the collection of mutiple points is strongly $\aleph_1$ compact is a z-compactification.*

COROLLARY 2.7: *Any compactification of a topological space in which the collection of multiple points is Lindelöf semi-stratifiable is a* z*-compactification.*

PROOF: Let M^* denote the closure of M in αX. Then M^* is a compactification of M and since M is separable, the weight of M^* is less than or equal to $2^{\aleph_0}$. Let $\mathscr{B}$ be an open basis for the topology of M^* such that card $(\mathscr{B}) \leqslant 2^{\aleph_0}$ and define

$$\mathscr{C} = \{(\mathrm{cl}_{\alpha X}(B_0), \mathrm{cl}_{\alpha X}(B_1)) | B_0, B_1 \in \mathscr{B} \text{ and } \mathrm{cl}_{\alpha X}(B_0) \cap \mathrm{cl}_{\alpha X}(B_1) = \emptyset\}.$$

For each $(\mathrm{cl}_{\alpha X}(B_0), \mathrm{cl}_{\alpha X}(B_1)) \in \mathscr{C}$, choose an $f \in C(\alpha X, I)$ such that $f(\mathrm{cl}_{\alpha X}(B_0)) = 0$ and $f(\mathrm{cl}_{\alpha X}(B_1)) = 1$. Let $\mathscr{F}$ denote the set of mappings obtained in this way and assume that $\mathscr{F}$ is most economically well-ordered (denote the order by $<$). Note that card $(\mathscr{F}) \leqslant 2^{\aleph_0}$. By transfinite induction we will construct for each $f \in \mathscr{F}$ a $\delta_f \in (0, 1)$ such that

(i) $\mathrm{cl}_{\alpha X}(f^{-1}[0, \delta_f] \cap X) \cap M = \mathrm{cl}_{\alpha X}(U(\delta_f, f)) \cap M$,

(ii) $\mathrm{cl}_{\alpha X}(U(\delta_f, f)) \cap \mathrm{cl}_{\alpha X}(V) \cap M = \mathrm{cl}_{\alpha X}(U(\delta_f, f) \cap V) \cap M$, for all $V \in \wedge \cdot \vee \cdot \{U(\delta_g, g) | g < f\}$.

Let f_0 be the first element of $\mathscr{F}$. Choose $\delta_{f_0} \in (0, 1)$ such that

$$\mathrm{cl}_{\alpha X}(f_0^{-1}[0, \delta_{f_0}] \cap X) \cap M = \mathrm{cl}_{\alpha X}(U(\delta_{f_0}, f_0)) \cap M.$$

Such a choice for δ_{f_0} is possible (lemma 2.3). Next, let $g \in \mathscr{F}$ and assume δ_f to be defined for all $f < g$ ($f \in \mathscr{F}$). Note that card $\{\delta_f | f < g\} < 2^{\aleph_0}$, since "$<$" is most economical. Define $\mathscr{V} = \wedge \cdot \vee \cdot \{U(\delta_f, f) | f < g\}$. Then if $V \in \mathscr{V}$,

$$\text{card } \{\delta \in (0, 1) | \mathrm{cl}_{\alpha X}(U(\delta, g)) \cap \mathrm{cl}_{\alpha X}(V) \cap M \neq \mathrm{cl}_{\alpha X}(U(\delta, g) \cap V) \cap M\} \leqslant \aleph_0$$

by lemma 2.4 and consequently

$$\text{card} \bigcup_{V \in \mathscr{V}} \{\delta \in (0, 1) | \mathrm{cl}_{\alpha X}(U(\delta, g)) \cap \mathrm{cl}_{\alpha X}(V) \cap M \neq \\ \neq \mathrm{cl}_{\alpha X}(U(\delta, g) \cap V) \cap M\} < 2^{\aleph_0}.$$

From lemma 2.3 it now follows that there exists a $\delta_0 \in (0, 1)$ such that for all

$$V \in \mathscr{V}: \mathrm{cl}_{\alpha X}(U(\delta_0, g)) \cap \mathrm{cl}_{\alpha X}(V) \cap M = \mathrm{cl}_{\alpha X}(U(\delta_0, g) \cap V) \cap M$$

and $\mathrm{cl}_{\alpha X}(g^{-1}[0, \delta_0] \cap X) \cap M = \mathrm{cl}_{\alpha X}(U(\delta_0, g)) \cap M$. Define $\delta_g = \delta_0$. This completes the construction of the $\delta_f (f \in \mathscr{F})$. Now define for each $f \in \mathscr{F}$ $H_f = f^{-1}[0, \delta_f] \cap X$. Notice that $H_f \in Z(X)$ for all $f \in \mathscr{F}$. Finally define $\mathscr{H} = \{H_f | f \in \mathscr{F}\}$ and

$$\mathscr{L} = \{Z \in Z(X) | \mathrm{cl}_{\alpha X}(Z) \cap M^* = \emptyset \text{ or } M^* \subset \mathrm{int}_{\alpha X} \mathrm{cl}_{\alpha X}(Z)\} \cup \mathscr{H}.$$

Using the compactness of αX it is easy to show that

$$\wedge \cdot \vee \cdot \{\mathrm{cl}_{\alpha X}(L) | L \in \mathscr{L}\}$$

is a separating ring. We will show that for each finite number of elements $L_0, L_1, \ldots, L_n \in \mathscr{L}$ the equality

$$(*) \qquad \mathrm{cl}_{\alpha X}\left(\bigcap_{i=0}^{n} L_i\right) = \bigcap_{i=0}^{n} \mathrm{cl}_{\alpha X}(L_i)$$

holds, which then proves our theorem.

If $L_i \notin \mathscr{H}$ $(i=0, 1, \ldots, n)$ then apply lemma 2.2 and use the analogous equality

$$(**) \qquad \mathrm{cl}_{\beta X}\left(\bigcap_{i=0}^{n} L_i\right) = \bigcap_{i=0}^{n} \mathrm{cl}_{\beta X}(L_i)$$

in βX. Note that equality (**) holds because $L_i \in Z(X)$ $(i=0, 1, \ldots, n)$. So it suffices to prove equality (*) in case $L_1, L_2, \ldots, L_n \in \mathscr{H}$ and $L_0 \notin \mathscr{H}$ (if all $L_i \in \mathscr{H}$ then enlarge $\{L_0, L_1, \ldots, L_n\}$ with $L_{n+1} = X$ and renumber them). Suppose that equality (*) does not hold; then there exists an

$$x_0 \in \bigcap_{i=0}^{n} \mathrm{cl}_{\alpha X}(L_i) \backslash \mathrm{cl}_{\alpha X}\left(\bigcap_{i=0}^{n} L_i\right).$$

We have to consider two cases:

case 1: $\mathrm{cl}_{\alpha X}(L_0) \cap M^* = \emptyset$.

Since

$$x_0 \in \bigcap_{i=0}^{n} \mathrm{cl}_{\alpha X}(L_i) \subset \mathrm{cl}_{\alpha X}(L_0)$$

it follows that $x_0 \notin M^*$. Let $Y = \alpha X \backslash M$. Notice that Y is homeomorphic to $\xi^{-1}(Y)$. As

$$x_0 \in \bigcap_{i=0}^{n} \mathrm{cl}_{\alpha X}(L_i) \cap Y = \bigcap_{i=0}^{n} \mathrm{cl}_{Y}(L_i) \text{ (lemma 2.1)}$$

$$= \mathrm{cl}_{\alpha X}\left(\bigcap_{i=0}^{n} L_i\right) \cap Y \subset \mathrm{cl}_{\alpha X}\left(\bigcap_{i=0}^{n} L_i\right),$$

this is a contradiction.

case 2: $M^* \subset \mathrm{int}_{\alpha X}\, \mathrm{cl}_{\alpha X}(L_0)$.

Let $L_i = f_i^{-1}[0, \delta_{f_i}] \cap X$ $(i=1, 2, \ldots, n)$.

If $x_0 \notin M$ then use the same technique as in case 1 in order to derive a contradiction. Next, suppose $x_0 \in M$; then

$$x_0 \in \bigcap_{i=1}^{n} \mathrm{cl}_{\alpha X}(f_i^{-1}[0, \delta_{f_i}]) \cap X) \cap \mathrm{cl}_{\alpha X}(L_0) \cap M$$

and consequently ((i))

$$x_0 \in \bigcap_{i=1}^{n} \mathrm{cl}_{\alpha X}\,(U(\delta_{f_i}, f_i)) \cap \mathrm{cl}_{\alpha X}\,(L_0) \cap M = \qquad ((\mathrm{ii}))$$

$$= \mathrm{cl}_{\alpha X}\,(\,\bigcap_{i=1}^{n} U(\delta_{f_i}, f_i)) \cap \mathrm{cl}_{\alpha X}\,(L_0) \cap M =$$

$$= \mathrm{cl}_{\alpha X}\,(\,\bigcap_{i=1}^{n} U(\delta_{f_i}, f_i)) \cap \mathrm{int}_{\alpha X}\,\mathrm{cl}_{\alpha X}\,(L_0) \cap M =$$

$$\subset \mathrm{cl}_{\alpha X}\,(\,\bigcap_{i=1}^{n} U(\delta_{f_i}, f_i) \cap \mathrm{int}_{\alpha X}\,\mathrm{cl}_{\alpha X}\,(L_0)) \cap M =$$

$$= \mathrm{cl}_{\alpha X}\,(\,\bigcap_{i=1}^{n} U(\delta_{f_i}, f_i) \cap \mathrm{int}_{\alpha X}\,\mathrm{cl}_{\alpha X}\,(L_0) \cap X) \cap M \subset$$

$$\subset \mathrm{cl}_{\alpha X}\,(\,\bigcap_{i=0}^{n} L_i) \cap M \subset$$

$$\subset \mathrm{cl}_{\alpha X}\,(\,\bigcap_{i=0}^{n} L_i),$$

which is a contradiction. □

Since separable metric spaces and countable spaces are Lindelöf semi-stratifiable we have the following corollaries:

COROLLARY 2.8 ([1], [9]): *Every metric compactification is a Wallman compactification.*

COROLLARY 2.9 ([11]): *Every countable multiple point compactification is a z-compactification.*

3. REGULAR WALLMAN SPACES

Let X be a topological space which is strongly $\aleph_1$ compact and let αX be a compactification of X such that the collection of mutiple points of αX is also strongly $\aleph_1$ compact. We will show that X is regular Wallman (a topological space is called *regular Wallman* if it is compact and possesses a separating ring consisting of regular closed sets. It is known that a regular Wallman space is a Wallman compactification of each dense subspace ([12])). If $B \subset X$, let $\bar{B}$ denote the closure of B in X. We need a simple lemma.

LEMMA 3.1: *Let U and V be open subsets of αX such that*

(i) $\overline{U \cap X} \cap \overline{V \cap X} = \overline{U \cap V \cap X}$
(ii) $\mathrm{cl}_{\alpha X}\,(U) \cap \mathrm{cl}_{\alpha X}\,(V) \cap M = \mathrm{cl}_{\alpha X}\,(U \cap V) \cap M$

then also

$$\mathrm{cl}_{\alpha X}\,(U) \cap \mathrm{cl}_{\alpha X}\,(V) = \mathrm{cl}_{\alpha X}\,(U \cap V).$$

PROOF: Suppose to the contrary there exists an

$$x_0 \in (\mathrm{cl}_{\alpha X}(U) \cap \mathrm{cl}_{\alpha X}(V)) \setminus \mathrm{cl}_{\alpha X}(U \cap V)).$$

Let $Y = \alpha X \setminus M$. Since X is hereditarily Lindelöf, every closed subset of X is a G_δ and consequently

$$\mathrm{cl}_{\beta X}(\overline{U \cap X}) \cap \mathrm{cl}_{\beta X}(\overline{V \cap X}) = \mathrm{cl}_{\beta X}(\overline{U \cap X} \cap \overline{V \cap X}) = \mathrm{cl}_{\beta X}(\overline{U \cap V \cap X}).$$

Hence it follows that

$$\mathrm{cl}_Y(U \cap X) \cap \mathrm{cl}_Y(V \cap X) = \mathrm{cl}_Y(U \cap V \cap X)$$

and therefore $x_0 \notin Y$. However it is clear that $x_0 \notin M$. Contradiction. □

THEOREM 3.2: *Any compactification of a strongly $\aleph_1$ compact space in which the collection of multiple points is also strongly $\aleph_1$ compact, is regular Wallman.*

COROLLARY 3.3: *Any compactification of a Lindelöf semi-stratifiable space in which the collection of multiple points is also Lindelöf semi-stratifiable, is regular Wallman.*

PROOF: Since X is separable it follows that the weight of αX is less than or equal to $2^{\aleph_0}$. Let $\mathscr{B}$ be an open basis for αX such that card $(\mathscr{B}) \leqslant 2^{\aleph_0}$. Define

$$\mathscr{C} = \{(\mathrm{cl}_{\alpha X}(B_0), \mathrm{cl}_{\alpha X}(B_1)) | B_0, B_1 \in \mathscr{B} \text{ and } \mathrm{cl}_{\alpha X}(B_0) \cap \mathrm{cl}_{\alpha X}(B_1) = \emptyset\}.$$

For each $(\mathrm{cl}_{\alpha X}(B_0), \mathrm{cl}_{\alpha X}(B)) \in \mathscr{C}$ choose an $f \in C(\alpha X, I)$ such that

$$f(\mathrm{cl}_{\alpha X}(B_0)) = 0 \text{ and } f(\mathrm{cl}_{\alpha X}(B_1)) = 1.$$

Let $\mathscr{F}$ denote the set of mappings obtained in this way and assume that $\mathscr{F}$ is most economically well-ordered (denote the order by $<$). Note that card $(\mathscr{F}) \leqslant 2^{\aleph_0}$. By transfinite induction we can construct, in a similar manner as in theorem 2.6, for each $f \in \mathscr{F}$ a $\delta_f \in (0, 1)$ such that

(i) $\mathrm{cl}_{\alpha X}(U(\delta_f, f)) \cap \mathrm{cl}_{\alpha X}(V) \cap M = \mathrm{cl}_{\alpha X}(U(\delta_f, f) \cap V) \cap M$
for all $V \in \wedge \cdot \vee \cdot \{U(\delta_g, g) | g < f\}$.
(ii) $\overline{U(\delta_f, f) \cap V \cap X} = \overline{U(\delta_f, f) \cap X} \cap \overline{V \cap X}$
for all $V \in \wedge \cdot \vee \cdot \{U(\delta_g, g) | g < f\}$.

Here we use lemma 2.4 in case $A = X$. From lemma 3.1 we deduce that

$$\wedge \cdot \vee \cdot \{\mathrm{cl}_{\alpha X}(U(\delta_f, f)) | f \in \mathscr{F}\}$$

is a separating ring of regular closed sets in αX. □

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